DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE
PERAMBALUR

DEPARTMENT OF SCIENCE AND HUMANITIES

QUESTION BANK

NUMERICAL METHODS-MA1251

UNIT-I

(SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS)

PART-A

1 .State the iterative formula for regula falsi method to solve f(x)=0

2. How to reduce the number of iterations while finding the root of an equation by Regula falsi method.

3. State the order of convergence and convergence condition for Newton Raphson method.

4. Write the iterative formula of Newton Raphson method.

5. State the principle used in Gauss Jordan method.

6. For solving a linear system, compare Gaussian elimination method and Gauss Jordan method.

7. Write a sufficient condition for Gauss seidel method to converge

8. Is the iteration method,a self -correcting method always?

9. Distinguish between direct and iterative (Indirect)method of solving Simultaneous equations.

10. Write the two methods to solve a system of equation

11. What is meant by diagonally dominant?

12. What is meant by self correcting method

13.When Gaussian elimination method fails ?

PART-B

1. Find the positive root of $x^3 - 2x - 5 = 0$ by Regula falsi method.

2. Find an approximate root of $x \log_{10} x - 1.2 = 0$ by Regula falsi method

3. Solve 3x-cosx =1 by Regula falsi method

4. Find by NR method ,the root of $\log_{10} x = 1.2$

5. By Newton Raphson find a non –zero root of $x^2 + 4 \sin x = 0$

6. Obtain Newton's iterative formula for finding N where N is a positive real number. Hence evaluate 142

7. Find the iterative formula for finding the value of 1 / N where N is a real number, using Newton Raphson method. Hence evaluate 1/26 correct to 4 decimal places

8. Find the negative root of $x^3 - \sin x + 1 = 0$ by using NR method

9.solve by iteration method $2x - \log_{10} x - 7$

10.Find a positive root of the equation $x^3 + x^2 - 100 = 0$

11. Solve the system of equations by Gauss elimination method

$$10x - 2y + 3z = 23$$
$$2x + 10y - 5z = -33$$
$$3x - 4y + 10z = 41$$

10. Solve the system of equations by Gauss Jordan method

$$2x + 3y - z = 5$$
$$4x + 4y - 5z = 3$$
$$2x - 3y + 2z = 2$$

13. Find the dominant eigen value and the corresponding eigen vector of the following matrix $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ **by power method.**

14. Find all the eigen value of the following matrix A= $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ **by power method.**

15.Find all the eigen values and eigen vectors of the matrix A=$\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$ **by jacobi's method.**

16.Using jacobi's method all the eigenvalues and eigenvectors of the matrix

A=$\begin{pmatrix} \frac{1}{\sqrt{2}} & 1 & \sqrt{2} \\ 1 & \frac{3}{\sqrt{2}} & 1 \\ \sqrt{2} & 1 & \frac{1}{\sqrt{2}} \end{pmatrix}$**.**

UNIT-II

(INTERPOLATION AND APPROXIMATION)

PART-A

1. **State Interpolation and Extrapolation.**
2. **State Newton's forward interpolation formula.**
3. **State Newton's backward interpolation formula.**
4. **State Gauss forward interpolation formula.**
5. **State Gauss backward interpolation formula.**
6. **State Newton's divided difference formula.**
7. **State Lagrange's Interpolation formula.**
8. **State Inverse Interpolation formula.**
9. **When will you use Newton's backward interpolation formula.**
10. **Using Lagrange's interpolation formula, find the polynomial for**

x:	**0**	**1**	**3**	**4**
y:	**-12**	**0**	**0**	**12**

obtain the interpolation quadratic polynomial for the given data by using Newton's forward difference formula.

11. **Find the parabola of the form $y = ax^2 + bx + c$ passing through the points (0,0) (1,1) and (2,20)**

12. **Write the Lagrange's fundamental polynomial $L_0(x)$ and $L_1(x)$ that satisfy the condition $L_0(x) + L_1(x) = 1$ for the data $[x_0, f(x_0)], [x_1, f(x_1)]$**

13. **State any two properties of divided difference.**

14. **What are the natural (or) free conditions in Cubic Spline.**

15. **Define natural spline.**

16. State the properties of cubic spline.

17. State the order of convergence of cubic spline.

18. State the error in N.F.I.F

19. State the error in N.B.I.F

20. What is the assumption we make when Lagrange's formula is used?

21. What advantage has Lagrange's formula over Newton?

22. What is disadvantage in practice in applying Lagrange's interpolation formula?

PART-B

1. From the following data estimate the number of persons earning weekly wages between 60 and 70 rupees.

Wages in Rs.	Below 40	40-60	60-80	80-100	100-120
No. of persons	250	120	100	70	50

2. For the given values evaluate f(9) using Lagrange's formula.

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

3. Using Newton's divided difference formula find u(3) given u(1) = -26,

u(2) = 12, u(4) =256 and u(6) = 844.

4. Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data.

Year	1997	1999	2001	2003
Profit in lakhs of Rs.	43	65	159	248

5. **Using Newton's forward interpolation formula, find the polynomial f(x) satisfying the following data. Hence find f(2)**

x	0	5	10	15
f(x)	14	379	1444	3584

6. **Obtain the root of f(x) = 0 by Lagrange Inverse Interpolation given that**

   f(30) = -30, f(34) = -13, f(38) = 3, f(42) = 18.

7. **Find f(x) as a polynomial in x for the following data by Newton's divided difference formula.**

x:	-4	-1	0	2	5
f(x):	1245	33	5	9	1335

8. **Find a polynomial of degree two for the data by Newton's forward difference method**

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

9. **Find f(8) by Newton's divided difference formula for the data:**

x:	4	5	7	10	11	13
f(x):	48	100	294	900	1210	2028

10. Find the polynomial f(x) by using Lagrange's formula and hence find f(3) for

x:	**0**	**1**	**2**	**5**
f(x):	**2**	**3**	**12**	**147**

11. Find f(x) as a polynomial in x for the following data by Newton's divided difference formula.

x:	**-4**	**-1**	**0**	**2**	**5**
f(x):	**1245**	**33**	**5**	**9**	**1335**

12. Using Newton's divided difference formula find f(x) and f(6) from the following data

x:	**1**	**2**	**7**	**8**
f(x):	**1**	**5**	**5**	**4**

13. From the following table, find the value of tan45º15′ by Newton's forward interpolation formula.

x º:	**45**	**46**	**47**	**48**	**49**	**50**
tan x º:	**1.00000**	**1.03553**	**1.07237**	**1.11061**	**1.15037**	**1.19175**

14. Fit the cubic spline for the data:

x:	**0**	**1**	**2**	**3**
f(x):	**1**	**2**	**9**	**28**

15. If f(0) = 0, f(1) = 0, f(2) = -12, f(4) = 0, f(5) = 600 and f(7) = 7308, find a polynomial that satisfies this data using Newton's divided difference interpolation formula. Hence, find f(6).

16. Given the following table, find f(2.5) using cubic spline functions:

i	**0**	**1**	**2**	**3**
x_i	**1**	**2**	**3**	**7**
$f(x_i)$	**0.5**	**0.3333**	**0.25**	**0.2**

17. The following values of x and y are given:

x:	**1**	**2**	**3**	**4**
y:	**1**	**2**	**5**	**11**

Find the cubic spline and evaluate y(1.5)

18. Use Lagrange's formula to fit a polynomial to the data:

x:	**-1**	**0**	**2**	**3**
y:	**-8**	**3**	**1**	**2**

and hence find y at x = 1

UNIT-III

(NUMERICAL DIFFERENTIATION AND INTEGRATION)

PART-A

1. **What is the condition for Simpson's 3/8 th rule and state the formula?**
2. **What is the condition for Simpson's 1/3 rd rule and state the formula?**
3. **Write down trapezoidal formula.**
4. **What is the order of error in the trapezoidal rule?**
5. **What is the order of error in the Simpson's 1/3 rd rule ?**
6. **What are the errors in the trapezoidal and Simpson's rules of numerical integration?**
7. **When can numerical differentiation be used?**
8. **Why Simpson's one-third rule is called a closed formula?**
9. **Write the formula for $\frac{dy}{dx}$ at $x \neq x_n$ using backward difference operator.**
10. **Write the formula for $\frac{dy}{dx}$ at $x=x_0$ using forward difference operator**
11. **Find $\frac{dy}{dx}$ at x = 1 from the following table**

x	1	2	3	4
y	1	8	27	64

12. **Using Newton's backward difference formula, write the formulae for the first and second order derivatives at the end values $x = x_n$ upto the fourth order difference term.**
13. **When does Simpson's rule give exact result?**
14. **Six sets of values of x and y are given (x's being equally spaced), write the formula to get $\int_{x_1}^{x_6} y\,dx$**
15. **What are the errors in Trapezoidal and Simpson's rule of numerical integration?**

16. **In order to evaluate $\int_{x_0}^{x_n} y dx$ by Simpson's 1/3 rule as well as by Simpson's 3/8 rule, what is the restriction on the number of intervals?**

17.**Using Trapezoidal rule evaluate $\int_{0}^{\Pi} \sin x dx$ by dividing the range into 6 equal parts.**

18. **Write down the trapezoidal rule to evaluate $\int_{1}^{6} f(x) dx$ with h = 0.5**

PART B

1. **Evaluate the integral $\int_{1}^{2}$ dx / 1+ x^2 using Trapezoidal rule with two sub intervals.**

2. **Dividing the range into 10 equal parts, fine the value of $\int_{0}^{\frac{\pi}{2}}$ sin x dx by**

   i) Trapezoidal rule ii)Simpson's rule.

3. **By dividing the range into ten equal parts, evaluate $\int_{0}^{\pi}$ sin x dx by**

   Trapezoidal rule and Simpson's rule.Verify your answer with integration.

4. **Evaluate $\int_{0}^{6}$ (dx /1+x^2) by i) Trapezoidal rule ii)Simpson's rule. Also check up the results by actual integration .**

5.**Evaluate $\int_{0}^{2}$ (dx/ x^2 +4) using Romberg's method . Hence obtain an approximate Value for π.**

6. Using Romberg's method evaluate $\int_{0}^{1}$ (dx/1+x) correct to three places of decimals.

8. Using three – point Gaussian quadrature formula, evaluate

i) $\int_{-1}^{1}$ $(1/ 1+x^2)$ dx

ii) $\int_{0}^{1}$ $(1 / 1+t^2)$ dt

9.Evaluate $\int_{0.2}^{1.5}$ $e^{-\ x2}$ dx using the three point Gaussian quadrature.

10(i)Evaluate $\int_{1}^{2}$ using Gauss 2 point formuka.

(ii)Evaluate $\int_{-1}^{1}(3x^2+5x^4)dx$ using gauss 2 point formula.

11.Evaluate $\int_{0.2}^{1.5} e^{-x^2}dx$ using the three point Gaussian quadrature.

10. Evaluate $\int_{0}^{2}\int_{0}^{2}$ f (x, y) dx dy by Trapezoidal Rule for the following data :

y/ x	0	0.5	1	1.5	2
0	2	3	4	5	5
1	3	4	6	9	11
2	4	6	8	11	14

11. Using Simpson's 1/ 3 rule evaluate $\int_{0}^{1}\int_{0}^{1}$ (1 / 1+ x+ y) dx dy taking h =k= 0.5

12. Evaluate $\int_1^2 \int_1^2$ (dx dy/ $x^2 + y^2$) numerically with h= 0.2 along x- direction and k= 0.25 along y- direction.

13. Compute $\int_4^{5.2}$ $\log_e x$ dx using Simpson's 1/ 3 and 3/8 rule.

14. Evaluate $\int_{-3}^{3}$ x^4 dx using (i)Trapezoidal rule (ii)Simpson's rule. Verify your results by actual integration.

15. The velocity v of a particle "s" from a point on its path is given by the table. Estimate the time taken to travel 60 feet by using Simpson's 1/ 3 rule

S(m)	0	10	20	30	40	50	60
V(m/s)	47	58	64	65	61	52	38

16. The table given below reveals the velocity v of a body during the time "t" speciied. Find its acceleration at t = 1.1

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.1	60.8

17. Find the first and second derivate of the function tabulated below at x = 0.6

x	0.4	0.5	0.6	0.7	0.8
y	1.5836	1.7974	2.0442	2.3275	2.6511

18.Evaluate $\int_0^1$ (dx/1+x) by two point Gaussian quadrature.

UNIT-IV

(INITIAL VALUE PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS)

PART-A

1. **State the disadvantage of Taylor series method.**

2. **Write down the fourth order Taylors Algorithm.**

3. **Which is better Taylor's method or R.K. Method?**

4. **State modified algorithm to solve $y' = f(x,y)$, $y(x_0) = y_0$ at $x = x_0 + h$.**

5. **Write down the Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = y_0$.**

5. **State the special advantage of Runge-Kutta method over Taylor series method.**

6. **Write down Adams-Bashforth predictor formula.**

7. **How many prior values are required to predict the next value in Adam'smethod?**

8. **What is a Predictor-Corrector method of solving a differential equation?**

PART-B

1. **Using Taylor series method find y at x=0.1 if $\frac{dy}{dx} = x^2y - 1$, $Y(0) = 1$.**

2. **Solve $y' = x + y$; $y(0)=1$ by Taylors series method. Find the values y at $x = 0.1$ and 0.2.**

3.Solve $\frac{dy}{dx}$ = y_2 + x_2 with y (0) =1. Use Taylor Series at x = 0.2 and 0.4.

Find x = 0.1.

4. i) Using Taylor Series method find y at x = 0.1correct to four decimal places from

$\frac{dy}{dx}$ = x^2- y, y (0) = 1, with h= 0.1. Compute terms upto x_4.

ii) Using Taylor's Series method , find y (1.1) given y' = x+y, y (1) = 0.

5. Using Taylor series method with the first five terms in the expansion find y (0.1) correct to three decimal places, given that $\frac{dy}{dx}$ = e^x –y^2, y(0) ==1.

6. i)By Taylor's series method find y (0.1) given that y '' = y+ xy' ,y(0) = 1,

y' (0) =0.

ii)Using Taylor's series method find y (1.1) and y (1.2) correct to four decimal places given y' = xy $^{1/3}$ and y(1) =1.

7. Using modified Euler's method solve; given that y'= 1-y, y (0) = 0 find y(0.1), y(0.2) and y (0.3).

8.Solve $\frac{dy}{dx}$ =$\log_{10}$(x+y), y(0) =2 by Euler's modified methgod and find the values of

y (.2), y (.4), and y (.6) by taking h=0.2

9. i) Using modified Euler's method solve; given that y' = y-x^2+1, y(0) = 0.5 find Y(0.2).

ii) Using modified Euler's method; find y(0.1), y (0.2); given

$\frac{dy}{dx}$ = x^2+y^2, y(0) =1.

10.Using improved euler's method find y(0.2)andy(0.4) from y'=x+y, y(0)=1 with h=0.2

11.using improved euler's method solve y'=x+y+xy, y(0)=1 compute y at x=0.1 by taking h=0.1

10. Compute y (0.2)and y (0.4) from y' = $\frac{y^2-x^2}{y^2+x^2}$ given that y (0)=1

11. Apply Runge – Kutta method to find approximate value of y for x =0.2

in steps of 0.1 if $\frac{dy}{dx}$ = x +y^2 given that y=1 when x =0.

12.Compute y (0.2) ,y (0.4) and y (0.6) ,given y' =x^3 + y, y (0) =2 by using Runge – Kutta fourth order method.

13.Given y ''- 2y' + 2y = e^{2x} sinx with y (0) = -0.4 and y' (0) = 0.6, using fourth order Runge – Kutta method find y(0.2).

14. Solve y' = 1/2 (1+ x^2)y^2, y (0) =1, y (0.1) = 1.06, y (0.2) = 1.12, y(0.3) = 1.21 compute y (0.4), using Milne's predictor corrector formula.

15. Solve 5xy'+y^2 =2, y (4) = 1, y (4.1) = 1.0049, y (4.2) = 1.0097, y (4.3) =1.0143, compute y (4.4) Milne's methods.

16. Given y' = xy + y^2, y (0) = 1.0, find y (0.1) by Taylors method y (0.2) by Euler's method y (0.3) by Runge – Kutta and y (0.4) by Milne's method.

17. **Solve $y' = x-y^2$, $0 \le x \le 1$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$, by Milne's method to find $y(0.8)$ and $y(1)$.**

18. **Compute the first 3 steps of the initial value problem $\frac{dy}{dx} = \frac{x-y}{2}$**

$y(0) = 1.0$ by Taylor series method and next step by Milen's method with step length

$h = 0.1$.

19. **Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$, $y(0.2) = 2.073$, $y(0.4) = 2.452$, $y(0.6) = 3.023$**

compute $y(0.8)$ by Milen's predictor – corrector method by $h = 0.2$.

20. **Given $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$,**

evaluate $y(1.4)$ by Adam's Bashforth method.

21. **Consider dy /dx = y- x2 +1, y (0) =0.5**

    i) using the modified Euler method find y (0.2)

    ii) using R-K method fourth order method find y (0.4) and y (0.6)

    iii)using Adam's Bashforth predictor- corrector method find y (0.8)

22. **Evaluate y (1.4): given y' = 1/x2 – y/x, y (1) =1, y (1.1) = 0.996, y (1.2) =0.986, y (1.3) = 0.972 by Adam's Bashforth formula.**

UNIT-V

(BOUNDARY VALUE PROBLEM IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS)

PART-A

1. **State the conditions for the equation . $Au_{xx}+Bu_{yy}+Cu_{yy}+Du_x+Eu_y+Fu=G$**

   Where A,B,C,D,E,F,G are function of x and y to be (i) elliptic (ii) parabolic(iii)hyperbolic

2. **State the condition for the equation $Au_{xx}+2Bu_{xy}+Cu_{yy}=f(u_x,u_y,x,y)$ to be**

   (i) elliptic (ii) parabolic(iii)hyperbolic when A,B,C are functions of x and y

3. **What is the classification of $f_x-f_{yy}=0$**
4. **What type of equations can be soled by using Crank-Nickolson's difference formula?**

5. **For what purpose Bender-Schmidt recurrence relation is used?**

Write a note on the stability and convergence of the solution of the difference

6. **What is the purpose of Liebmann's process?**

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7. **Define a difference quotient.**

PART – B

1. **Solve the Poisson's equation** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-10(x^2+y^2+10)$ **over the square with sides *x=0=y, x=3=y* with *u=0* on the boundary and mesh length is 1.**

2. **Solve the Poisson equation** $u_{xx}+u_{yy}=-81xy, 0<x<1, 0<y<1$ **given that *u(0,y)=0, u(x,0)=0, u(1,y)=100, u(x,1)=100* and *h*=1/3.**

3. **Solve** $u_{xx} + u_{yy} = 0$ **,** $0 \le x,\ \ y \le 1$ **with *u(0,y)=10=u(1,y)* and *u(x,0)=20=u(x,1)*.**

   Take h=0.25 and apply Liebmann method to 3 decimal accuracy.

4. **Solve** $y_{tt} = y_{xx}$ **upto t=0.5 with a spacing of 0.1 subject to y(0,t)=0, y(1,t)=0,**

   y_t(x,0)=0 and y(x,0)=10+x(1-x)

5. **Solve** $32u_t = u_{xx},\ 0 < x < 1,\ \ t > 0,\ \ \ u(x,0) = 0,\ \ u(0,t) = 0,\ \ \ u(1,t) = t$ **choose**

   h=0.25

6. **Using Crank-Nickolson's Implicit scheme, Solve**

   $16u_t = u_{xx},\ \ 0 < x < 1,\ \ t > 0,$ **given that u(x,0)=0, u(0,t)=0, u(1,t)=100t**

7. **Approximate the solution to the following elliptic partial differential equation**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{xy}(x^2 + y^2), 0 < x < 1, 0 < y < 1, u(0,y) = 1, u(1,y) = e^{y}, 0 \le y \le 1 \,\&\, u(x,0) = 1, u(x,1) = e^{x},$$

$$0 \le x \le 1,\ \ \ u\sin g\ \ h = k = \frac{1}{3}$$